자원 · 환경경제연구 제 26 권 제 4 호
Environmental and Resource Economics Review
Volume 26, Number 4, December 2017 : pp. 613~639
DOI: https://doi.org/10.15266/KEREA.2017.26.4.613

A Comparative Analysis of Maximum Entropy and Analytical Models for Assessing Kapenta (*Limnothrissa miodon*) Stock in Lake Kariba†

Itai Hilary Tendaupenyu* and Hee-Dong Pyo**

ABSTRACT : A Maximum Entropy (ME) Model and an Analytical Model are analyzed in assessing Kapenta stock in Lake Kariba. The ME model estimates a Maximum Sustainable Yield (MSY) of 25,372 tons and a corresponding effort of 109,731 fishing nights suggesting overcapacity in the lake at current effort level. The model estimates a declining stock from 1988 to 2009. The Analytical Model estimates an Acceptable Biological Catch (ABC) annually and a corresponding fishing mortality (F) of 1.210/year which is higher than the prevailing fishing mortality of 0.927/year. The ME and Analytical Models estimate a similar biomass in the reference year 1982 confirming that both models are applicable to the stock. The ME model estimates annual biomass which has been gradually declining until less than one third of maximum biomass (156,047 tons) in 1988. It implies that the stock has been overexploited due to yieldings over the level of ABC compared to variations in annual catch, even if the recent prevailing catch levels were not up to the level of MSY. In comparison, the Analytical Model provides a more conservative value of ABC compared to the MSY value estimated by the ME model. Conservative management policies should be taken to reduce the aggregate amount of annual catch employing the total allowable catch system and effort reduction program.

Keywords : Maximum sustainable yield, Acceptable biological catch, Maximum entropy, Overexploitation, Mortality

JEL 분류 : C61, Q22, Q57

Received: August 16, 2017. Revised: November 30, 2017. Accepted: December 5, 2017.

† The paper was supported by Pukyong National University (CD 2016-1268). I greatly appreciate the constructive comments and suggestions of anonymous reviewers.

* Graduate Student, Pukyong National University, First author(e-mail: itaihilary@yahoo.co.uk)
** Professor, Pukyong National University, Corresponding author(pyoh@pknu.ac.kr)

카리브호수 카펜타 자원량 추정을 위한 최대엔트피모델과 분석적 모델의 비교분석†

이타이 텐다우펜유* · 표희동**

요 약 : 카리브호수의 카펜타 자원량을 추정하기 위해 최대엔트로피(ME)모델과 분석적 모델이 적용된다. ME모델을 이용하여 25,372톤의 최대지속가능 어획량(MSY)과 MSY의 어획노력량인 109,731의 어획일수(fishing nights)를 추정하였는데, 이는 현재 어획노력량 수준이 과잉투자됨으로써 1988년 이후 2009년 현재까지 자원량을 감소시키는 요인인 것을 나타낸다. 분석적 모델은 매년의 생물학적 허용 어획량(ABC)과 연간 1.21의 어획사망계수(일반적 어획사망계수인 0.927 보다 큰)를 추정한다. 이 두 모델은 1982년 기준년도의 자원량 추정에 적용할 수 있는 유사한 자원량을 추정한다. ME모델에 의하면 1988년의 최대 자원량(156,047톤)에 대해 1/3수준이하 까지 점점 하락하는 결과를 추정하였는데, 이는 최근의 어획량이 MSY 수준 이하이지만 ABC수준보다 높게 나타나 남획된 것을 암시한다. 다시 말해서, 분석적 모델은 ME모델에서의 MSY보다 더 보수적인 ABC를 제공함으로써, 보수적인 어업관리정책(총허용어획량제도, 어획노력감소정책 등)을 적극적으로 고려해야함을 내포하고 있다.

주제어 : 최대지속가능어획량, 생물학적 허용어획량, 최대엔트로피, 남획, 사망계수

접수일(2017년 8월 16일), 수정일(2017년 11월 30일), 게재확정일(2017년 12월 5일)

† 이 논문은 부경대학교 산학협력단(CD 2016-1268)에 의해 지원되었습니다. 건설적인 논평과 제안을 주신 익명의 심사자들에게 진심으로 감사드립니다.

* 부경대학교, 대학원졸업, 주저자(e-mail: itaihilary@yahoo.co.uk)

** 부경대학교, 교수, 교신저자(e-mail: pyoh@pknu.ac.kr)

Ⅰ. Introduction

Lake Kariba is a vast inland water body shared by the Republics of Zimbabwe and Zambia with a major commercial fishery of the introduced clupeid popularly known as Kapenta. After its introduction in the late 1960's, Kapenta led to the development of a commercial catch that stands as a vital food security item for both nations. Resource managers have developed regulatory methods to manage Kapenta stocks which include fishing gear restrictions, closed areas for fishing and the issuance of licenses to protect the fishery resource. However, after 1999 it was been observed that catches were on a decline leading to the need for a proper analysis of fish stocks and the formulation of management measures that ensure optimal, yet sustainable, utilization of the strategic food item. Catch and effort data are well recorded but data on growth parameters is relatively poor and was carried out only in the early years of the fishery.

Maximum sustainable yield (MSY) is the best known proxy for sustainability and is defined as the greatest level of catch that can be achieved whilst maintaining resource sustainability. In the absence of specific information on age and growth of a fishery, the most commonly applied alternatives to fisheries stock assessment techniques are commonly referred to as Surplus Production Models (SP models). The maximum entropy (ME) model developed by Golan et al. (1996a, 1996b) can be used to overcome several limits on the SP model. These can be applied to estimate yearly fisheries stock, MSY, and the maximum sustainable biomass using non-linear programming (Pyo, 2006). An alternative approach to stock assessment is that of using growth parameters. From these, the Acceptable Biological Catch can be estimated which is a more conservative reference point compared to MSY. Dynamic Pool models can be applied to estimate the parameter and in this analysis the Zhang and Megrey (2010) Model will be assessed.

Previous studies in Kapenta stock assessment were carried out by Cochrane (1984) and Marshall (1987) but there was little reference on biological reference points in their researches. The Maximum Entropy Model has been applied in studies by Golan (1996) as well as by Pyo (2006) on estimating anchovy stock in Korea. Other studies using the Maximum Entropy include Hilborn and Walters (1987), Vignaux et al. (1998), Xiao (1997). The Analytical Model by Zhang and Megrey (2010) is a fairly new model which uses growth parameters of the fish species to come up with biological parameters that aid in stock assessment. Due to the absence of data on growth parameters in literature for Kapenta, the only available complete data on growth parameters for the fishery was that from studies done by Cochrane (1984) in 1982.

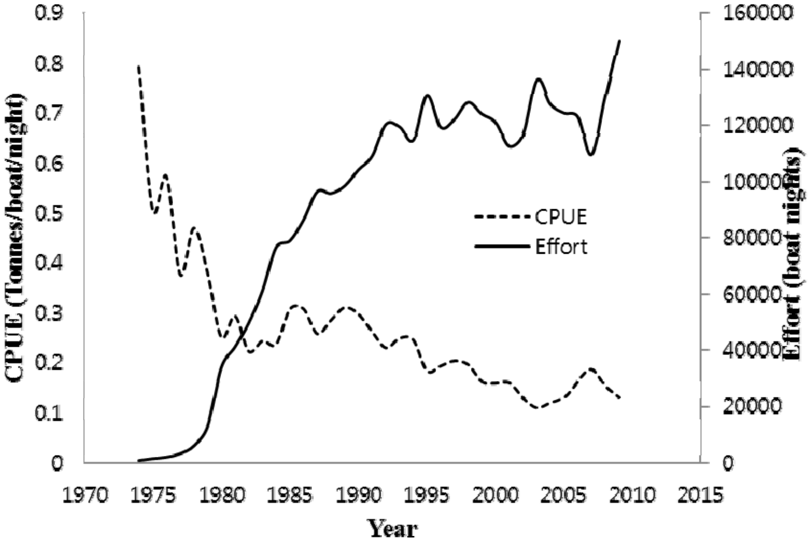
There is a need to conduct population modeling of the fishery under a data deficient situation. The objective of this study is to conduct a comparative analysis of the fishery using an Analytical Model (Dynamic Pool Model) and a Maximum Entropy Model (Surplus Production Model) and ultimately come up with the most accurate biological reference points that aid in decision making towards the sustainability of the fishery.

II. Methods

1. Catch and Effort data

Data on catch and effort have been extensively collected for the fishery since its inception in 1974 with monthly returns submitted by fishermen to the regulatory authorities in both countries. Lake Kariba Fisheries Research Institute provided data for the Zimbabwean fishery whilst the Zambian fishery data was obtained from the relevant Ministry. A complete data set from 1974 to 2009 is analyzed. Table 1 below shows the catch statistics for the Kapenta fishery since its inception in 1974. The time series for catch and catch per unit effort (CPUE) shows a pattern where initially catch rates were high and effort levels were low. Catch however falls as effort levels rise and the stock is depleted. Pascoe (1998) states that effort and CPUE are highly correlated. The figure below shows the CPUE and catch trends in the fishery. It is observed that effort levels are increasing since the inception of the fishery and on the other hand CPUE has been on the decline.

〈Figure 1〉 Kapenta Catch and CPUE trends in Lake Kariba



〈Table 1〉 Kapenta catch and effort statistics

	Zimbabwe		Zambia		Combined		
	Catch (tonnes)	Effort (nights fished)	Catch (tonnes)	Effort (nights fished)	Catch (tonnes)	Effort (nights fished)	CPUE
1974	487	615			487	615	0.79
1975	654	1,294			654	1,294	0.51
1976	1,050	1,833			1,050	1,833	0.57
1977	1,171	3,111			1,171	3,111	0.38
1978	2,772	5,903			2,772	5,903	0.47
1979	4,874	12,847			4,874	12,847	0.38
1980	8,395	33,516			8,395	33,516	0.25
1981	12,006	40,935			12,006	40,935	0.29
1982	8,450	37,776	2,601	11,686	10,989	49,462	0.22
1983	8,548	38,865	6,227	22,083	14,830	60,948	0.24

〈Table 1〉 Kapenta catch and effort statistics

	Zimbabwe		Zambia		Combined		
	Catch (tonnes)	Effort (nights fished)	Catch (tonnes)	Effort (nights fished)	Catch (tonnes)	Effort (nights fished)	CPUE
1984	10,394	41,234	7,702	35,236	18,106	76,470	0.24
1985	14,586	41,403	9,360	37,378	24,179	78,781	0.31
1986	15,747	45,790	10,449	40,520	26,543	86,310	0.31
1987	15,823	52,414	8,994	43,933	24,818	96,347	0.26
1988	18,366	53,403	8,907	42,296	27,272	95,699	0.28
1989	20,112	54,919	10,409	43,440	30,521	98,359	0.31
1990	21,758	59,193	9,185	44,938	30,942	104,131	0.30
1991	19,306	62,208	9,258	46,819	28,564	109,027	0.26
1992	18,931	71,066	8,658	49,259	27,599	120,325	0.23
1993	19,957	68,155	9,722	51,231	29,680	119,386	0.25
1994	19,232	71,249	8,910	43,462	28,142	114,711	0.25
1995	15,280	75,443	8,674	55,381	23,954	130,824	0.18
1996	15,423	73,524	7,593	45,693	23,016	119,217	0.19
1997	17,034	75,633	7,813	46,436	24,847	122,069	0.20
1998	15,288	74,770	9,822	53,475	25,110	128,245	0.19
1999	11,208	64,091	8,955	59,960	20,163	124,051	0.16
2000	10,500	65,625	8,863	55,394	19,363	121,019	0.16
2001	9,500	59,375	8,500	53,125	18,000	112,500	0.16
2002	7,150	55,000	8,000	61,538	15,150	116,538	0.13
2003	7,500	68,182	7,481	68,009	14,981	136,191	0.11
2004	8,735	72,792	6,574	54,784	15,309	127,576	0.12
2005	10,158	78,138	6,251	46,256	16,409	124,394	0.13
2006	12,503.04	78,144	7,659	44,926	20,162	123,070	0.16
2007	10,940.16	78,144	9,476	31,421	20,416	109,565	0.19
2008	12,157.2	81,048	7,860	49,258	20,017	130,306	0.15
2009	9,727.974	87,384	9,993	62,948	19,721	150,332	0.13

2. Maximum Entropy Model

1) Formulation of the ME model

Difficult dynamic problems are faced under the conventional estimation rules which are that; an ill-posed problem that the number of parameters to be estimated exceeds the number of observations and an underdetermined or under-identified problem which cannot be alleviated by obtaining more data.

Given probabilities p_i such that $\sum p_i = 1$ for random variables (X_i), Shannon (1948) defined the entropy as a measure of uncertainty in the probability that maximizes

$$H(p) = -\sum p_i \ln p_i \quad (1)$$

Subject to data consistency (available evidence points) in the form of J moment conditions

$$\sum p_i x_{ij} = a_j,\ j = 1, 2, \ldots\ldots,\ J \quad (2)$$

And normalization-additivity (adding up) constraint

$$\sum p_i = 1 \quad (3)$$

where $J<N$. The ME model seeks to make the best predictions possible from the limited data information that is available, transforming the empirical moments into the probability distribution representing our state of knowledge (Golan et al. 1996b).

2) ME model for stock assessment of Kapenta

For a ME model of fish stock assessment, fisheries production function can be formularized using a Cobb-Douglas production and logistic growth function as follows:

$$C_t = AE_t^{\alpha}X_t^{\beta}\exp(\varepsilon_t) \tag{4}$$

$$X_{t+1} = [X_t + rX_t(1 - \frac{X_t}{K}) - C_t]\exp(\mu_t) \tag{5}$$

α and β are parameters representing the effort and stock elasticity respectively, and ε_t and μ_t are error terms for C and X at time t, respectively. The above functions can be converted to log form as follows:

$$\ln C_t = \ln A + \alpha \ln E_t + \beta \ln X_t + \varepsilon_t \tag{4'}$$

$$\ln X_{t+1} = \ln X_t + \ln S_t + \mu_t \tag{5'}$$

where

$$S_t = 1 + r(1 - \frac{X_t}{K}) - \frac{C_t}{X_t}$$

In this formulation the observable variables are C_t and E_t and the parameters to be internally derived from the formulation are the probability distributions of A, α, β, r, X_t, K, ε_t and μ_t. Therefore the formulations are involved in an ill-posed problem as they have much more parameters estimated than observed variables. Furthermore, there is a method to impose prior restrictions on the parameter estimates by spanning the possible parameter range for each parameter. For example, if A, α and β are believed that they range between 0 and 1, they will be specified by a tri-uniform distribution such as [0, 0.5, 1].

$$A = p_1^A \times 0 + p_2^A \times 0.5 + p_3^A \times 1 \tag{6}$$

$$\alpha = p_1^{\alpha} \times 0 + p_2^{\alpha} \times 0.5 + p_3^{\alpha} \times 1 \quad (7)$$

$$\beta = p_1^{\beta} \times 0 + p_2^{\beta} \times 0.5 + p_3^{\beta} \times 1 \quad (8)$$

In such context, limited prior information for r and K can be imposed by using the estimates from SP model[1)] as follows:

$$r = p_1^{r} \times 0 + p_2^{r} \times \frac{m}{2} + p_3^{r} \times m \quad (9)$$

$$K = p_1^{K} \times 0 + p_2^{K} \times \frac{n}{2} + p_3^{K} \times n \quad (10)$$

$$X_t = p_{t1}^{X} \times 0 + p_{t2}^{X} \times \frac{h}{2} + p_{t3}^{X} \times h \quad (11)$$

$$\varepsilon_t = p_{t1}^{\varepsilon} \times (-e) + p_{t2}^{\varepsilon} \times 0 + p_{t3}^{\varepsilon} \times (+e) \quad (12)$$

$$\mu_t = p_{t1}^{\mu} \times (-e) + p_{t2}^{\mu} \times 0 + p_{t3}^{\mu} \times (+e) \quad (13)$$

where *m*, *n* and *h* stand for upper bounds of *r*, *K* and X_t respectively, and *e* is specified to be symmetric around zero for ε_t and μ_t.

In conclusion, the generalized stochastic non-linear ME for stock assessment of Kapenta in Lake Kariba can be structured in scalar-summation notation, using a criterion with non-negative probability factors, as

1) Guided by prior assessment using Surplus Production models, estimated values can be inputted into the model. As explained in section 4 below, this value is adjusted so as to come up with a more accurate analysis of the fishery.

$$\text{Max}\ [-\textstyle\sum_{i=1}^{3} p_i^j - \sum_t \sum_{i=1}^{3} p_{ti}^k \ln p_{ti}^k] \tag{14}$$

Subject to the data consistency with (4’), (5’), (6), (7), (8), (9), (10), (11), (12), (13) in which m, n, h and e are replaced by 2, 185000 and 92500[2)] and 0.3, respectively, and the adding up constraints:

$$\textstyle\sum_i^3 p_i^j = 1,\ \sum_i^3 p_{ti}^x = 1,\ \sum_{ti}^3 p_i^\varepsilon = 1, \sum_i^3 p_{ti}^\mu = 1 \tag{15}$$

where

j = A, α, β, r, K and k = X, ε and μ, and t = 1, 2, 3 ⋯. n-1.

This formulation is a general non-linear inversion procedure for recovering both time variant parameters. These estimates may also be used for defining measures of uncertainty and precision for fish stock assessment (Golan et al., 1996a).

3. Analytical Model

1) Biological Parameters

(1) Length weight parameters

Parameters for length and weight are obtained from the Fishbase website together with data for 1982 length-weight frequencies. This data enabled the conversion of length to weight data using the equation: W = αL^β. The parameters α and β for *Limnothrissa miodon* are found to be 0.01 and 2.86 respectively. The value of β shows that the species has an isometric growth according to Tresierra and Culquichicón (1993).

2) The values were obtained by selecting the values of K and $\frac{K}{2}$ which provided the least Mean square error value between observed and estimated catch in simulations in the ME model. These values would then be used as the reserve input in the model.

(2) Growth Parameters

Several studies on the growth of *Limnothrissa miodon* were carried out during the 1982-1992 period by Cochrane (1984), Marshall (1987) and Chifamba (1992). Cochrane's (1984) von Bertalanffy growth parameters were selected and used in the analysis as they fitted well with length-frequency data for the 1982 analysis. The parameters referred to are: the asymptotic length (L_∞) = 8.1cm, the estimated growth coefficient (*K*) = 1.74 and t_0 = -0.13. L_∞ is the asymptotic length; *K* is the growth coefficient and t_0 age of fish when the size is zero. The parameters are substituted into the von Bertalanffy equation, which is expressed as follows:

$$L_t = L_\infty (1 - e^{-k(t - t_0)}) \tag{16}$$

2) Mortality

(1) Natural Mortality

Natural mortality was calculated using the equation by Zhang and Megrey (2006) which is expressed as a function of the growth coefficient (*K*), the power parameter of the weight and length relationship (β), the age of fish when the size is zero (t_0), and the critical age (t_{mb}).

$$M = \frac{\beta K}{e^{K(t_{mb} - t_0)} - 1} \tag{17}$$

where $t_{mb} = C_i \cdot t_{\max}$. Here t_{max} is the maximum age observed in the population (Beverton and Holt, 1959; Zhang and Megrey, 2006), and C_i is the constant for specific ecological groups, demersal species (0.440), pelagic species (0.302) and the overall mean is (0.393). The species under investigation is a pelagic species therefore the value of 0.302 was considered in estimating natural mortality.

(2) Fishing Mortality

The method used to calculate fishing mortality was through the equation proposed by Zhang and Megrey (2010). This method is expressed as a function of the biomass by length (B_{li}) to biomass ($B_{li+\triangle t}$) , the natural mortality (M), the time needed to grow from length-class l_i to length class $l_{i+\Delta t}$, and weight by length (G_{li}):

$$F_{li} = \frac{\log_e\left(\frac{B_{li}}{B_{li+\triangle t}}\right) - M \times \triangle t_{li+} G_{li}}{\triangle t_{li}} \tag{18}$$

and was estimated the weighted fishing mortality by the equation:

$$Weighted\ F_{li} = \frac{\sum B_{li} F_{li}}{\sum B_{li}} \tag{19}$$

(3) Total Mortality

The total mortality (Z) was estimated by calculating the sum of natural mortality (M) and fishing mortality (F).

$$Z = M + F \tag{20}$$

3) Biomass

The combined catch for 1982 of 10,989 tons was used in the estimation of the total biomass for the species. A biomass based length cohort analysis model by Zhang and Megrey (2010) was used in the estimation of biomass in this analysis. Five essential pieces of information were required, to carry out this task, which were as follows:

- One year of length composition data for the catch;

- Weight of catch for each length-class (l_i);
- Estimate of Natural mortality (M)
- von Bertalanffy Growth parameters (K, t_0, and L_∞);
- Allometric parameters relating length to weight (α and β)

Five steps were followed according to the Zhang and Megrey (2010) method to achieve the desired result:

Step 1: Calculation of weight from length for each length-class (l_i) using the allometric weight equation.

$$W_{li} = \alpha \times l_i^{\beta} \tag{21}$$

Step 2: Calculation of G_{li} to convert length to weight to calculate G per length class using the follow equation.

$$G_{li} = \log_e\left(\frac{w_{li+\triangle l}}{w_{li}}\right) \tag{22}$$

where, l_i is length-class, $l_i + \triangle l_i$ represent the time needed to grow from length class l_i to length class $l_i + \triangle l_i$.

Step 3: Δt - the time needed to grow from length class l_i to length class $l_i + \triangle l_i$, calculated for each length-class (l_i):

$$\triangle t_{li} = \frac{1}{k} log_e\left(\frac{L_\infty - l_i}{L_\infty - l_{i+\triangle l}}\right) \tag{23}$$

Step 4: Population biomass in the longest length-class (l_i) is estimated based on the biomass-based catch equation and the estimate of F_t.

$$B_{l_i} = C_{l_i} \times \frac{(M + F_T) \times \Delta t_{l_i} - G_{l_i}}{F_T \times \Delta t_{l_i}} \tag{24}$$

where, F_T is assumed to be equal to 0.5M for a lightly exploited stock, M for a moderately exploited stock, or 2M for a heavily exploited stock. *Limnothrissa miodon* is considered a heavily exploited stock. C_{l_i} denotes the total catch by weight by length-class (l_i).

Step 5: Involves the progression from the longest length-class to the smallest length-class (l_i) to calculate B_{l_i} using the follow equation:

$$B_{li} = B_{li+\Delta l} \exp\left[\frac{M}{K} log_e\left(\frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}}\right) - G_{li}\right] + C_{li} \exp\left[\frac{M}{2K} log_e\left(\frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}}\right) - \frac{G_{li}}{2}\right] \tag{25}$$

4) Acceptable Biological Catch (ABC)

Due to lack of specific reference points to ensure that a fish species is not exploited to unsustainable levels many fisheries around the world are in danger of collapse. Concerted efforts have been made by resource managers to set these biological reference points (BRP's) using available information on the fishery. The BPRs are usually fishing mortalities (F) or abundance levels (thresholds).

One of major limitations of BRPs based on yield per recruit such as F_{max} is that the effects on the spawning population are essentially ignored. As a worst case scenario, suppose that infinite fishing pressure were applied at critical age *t* but that fish matured at ages older than *t*. The maximum yield per recruit would be taken, but at the expense of rendering the population extinct. The class of BRPs coming out of this approach is denoted $F_{x\%}$, where is generally in the range of 20%-40%. Reference fishing mortality ($F_{x\%}$) result in a spawning stock biomass or egg production per recruit that is *x%* of that with no fishing (Quinn and Deriso, 1999).

Quinn and Szarzi (1993), cited by Quinn and Deriso (1999) suggested that fishing mortalities between $F_{30\%}$ and $F_{45\%}$ in terms of spawning abundance instead of spawning biomass would result in sustainable harvests. The information used to estimate the $F_{x\%}$ was: length class, weight at length relationship, maturity rate, selectivity at length and mortality at length (Zhang and Megrey, 2010).

$$F_{x\%} = \frac{\sum_{l=i}^{l_\lambda} B_i^l \cdot\ m_i \cdot\ e^{G_i-(M+F_{40\%} \cdot S_i)\left(\frac{1}{K} ln\left(\frac{(L_\infty - l_i)}{(L_\infty - l_{i+1})}\right)\right)}}{\sum_{l=i}^{l_\lambda} B_i^l \cdot\ m_i \cdot\ e^{G_i - M\left(\frac{1}{K} ln\left(\frac{(L_\infty - l_i)}{(L_\infty - l_{i+1})}\right)\right)}} \tag{26}$$

where, m_i is the maturity rate by length *i*, *M* is natural mortality, S_i is selectivity at length *I*, $B_i^{'}$ number of population at length *i*, *K* is growth coefficient of von Bertalanffy parameter, L_∞ is asymptotic length, G_i is growth coefficient by weight at length *i* and l_i length class.

If F = 0, $B_i = B_{i-1} \cdot\ e G_{i-1} - M \cdot\ \triangle_{i-1} = B_{i-1} \cdot\ \sum_{l=i}^{l_\lambda} B_i^l \cdot\ m_i \cdot\ e^{G_{i-1} - M\left(\frac{1}{K} ln\left(\frac{(L_\infty - l_i)}{(L_\infty - l_{i+1})}\right)\right)}$

If F =x%, $B_i^{'} = B_{i-1}^{'} \cdot e^{G_i - (M + F_{40\%} \cdot S_{i-1})\left(\frac{1}{K} ln\left(\frac{(L_\infty - l_i)}{(L_\infty - l_{i+1})}\right)\right)}$

In this study to estimate ABC, x% stands for 40%

$$G_i = \ln\left(\frac{w_{i+1}}{w_i}\right) \tag{27}$$

$F_{40\%}$ of the level of biomass ($B_{40\%}$) was estimated by the equation:

$$\mathrm{B}_{40\%} = \mathrm{B}_{\mathrm{c}} \times \frac{\frac{\mathrm{SB}}{\mathrm{R}} | \mathrm{F}_{40\%}}{\frac{\mathrm{SB}}{\mathrm{R}} | \mathrm{F}_{\mathrm{C}}} \tag{28}$$

where, B_c is the current biomass, $\frac{SB}{R}|F_{40\%}$ is the spawning biomass per recruit with $F_{40\%}$ and $\frac{SB}{R}|F_c$ is the spawning biomass per recruit with current F.

Subsequently this information was analyzed with Acceptable Biological Catch (ABC), which provides an acceptable level of capture of a species or species group. Based on the information available, Tier methods for ABC were used in the analysis. The estimates of ABC are compared to variations in annual catch to investigate the reason for decline of Kapenta (*Limnothrissa miodon*) stock in Lake Kariba. Tier 4 method is employed to estimate ABC.

〈Table 2〉 Methods used to determine ABC (modified from Zhang and Lee (2001))

Tier 1. Information available: Reliable estimates of annual B and F, B_{MSY}, F_{MSY}, $F_{X\%}$, M and environmental factor.
1a) Stock status: $B/B_{MSY} > 1$: $F_{ABC} = F_{MSY}$ 1b) Stock status: $\alpha < B/B_{MSY} \leq 1$: $F_{ABC} = F_{MSY} \times (B/B_{MSY}-\alpha)/(1-\alpha)$ 1c) Stock status: $B/B_{MSY} \leq \alpha$: $F_{ABC}=0$
Tier 2. Information available: Reliable estimates B, $B_{X\%}$ and $F_{X\%}$ 2a) Stock status: $B/B_{40\%} > 1$: $F_{ABC} = F_{40\%}$ 2b) Stock status: $\alpha < B/B_{40\%} \leq 1$: $F_{ABC} = F_{40\%} \times (B/B_{40\%}-\alpha)/(1-\alpha)$ 2c) Stock status: $B/B_{40\%} \leq \alpha$: $F_{ABC}=0$
Tier 3. Information available: Reliable estimates B and $F_{0.1}$: $F_{ABC} = F_{0.1}$
Tier 4. Information available: Time-series catch and effort data. 4a) Stock status: $B/B_{MSY} > 1$: ABC=MSY 4b) Stock status: $\alpha < B/B_{MSY} \leq 1$: ABC=MSY $\times (B/B_{MSY} - \alpha)/(1-\alpha)$ 4c) Stock status: $B/B_{MSY} \leq \alpha$: ABC =0
Tier 5. Information available: Reliable catch history. ABC=M $\times$ Y_{AM} (arithmetic mean catch over an appropriate time period), $0.5 \leq P \leq 1.0$
i) Equation used to determine ABC in tiers 1-3: ABC= $\frac{BF_{ABC}}{M+F_{ABC}}(1-e^{-(M+F_{ABC})})$ where B_i: *Biomass at age i M: instantaneous coefficient of actual mortality,* F_{ABC}*: instantaneous coefficient of fishing mortality for ABC determined by the data available and the stock status, r: recruit age,* t_L*: maximum fishing age.* ii) For tiers 1, 2 and 4, α is set at a default value of 0.05.

Spawning stock biomass-per-recruit (SBPR) analysis was conducted. Reference points of F and SBPR for a percentage of maximum spawning potential are calculated. When $F=0$, the spawning biomass per recruit (SB/R) is,

$$\frac{SB}{R}=\sum_{t=tr}^{t_{\lambda}} m_t \cdot e^{-M(t_c-tr)} \cdot e^{(M+F)(t-t_c)} \cdot W_{\infty}(1-e^{K(t-t_0)}) \tag{29}$$

where m_t is the maturity rate by time, K is growth coefficient, t_0 is the age of fish when the size is zero, t_c is age of first capture, t_r is the recruitment age, W_{∞} is the asymptotic weight, F is the fishing mortality, M is the natural mortality.

4. Determining accuracy of a model

Forecasts are usually produced for the whole out of sample period, which would then be compared to the actual values, and the difference between them aggregated in some way. The forecast error for observation, i is defined as the difference between the actual value for the observation i and the forecast made for it.

1) Mean Square Error

Denoting s steps ahead forecasts of a variable made at a time t as $f_{t,s}$ and the actual value of the variable at time t as $Y_{t,}$, then the mean square error can be defined as:

$$\text{MSE}=\frac{1}{T-(T_1-1)}\sum_{t=T_1}^{T}(y_{t+s}-f_{ts})^2 \tag{30}$$

where T is the total sample size (in sample + out of sample), and T_1 is the first out of sample forecast observation. Thus in sample model estimation initially runs from observation 1 to ($T_1 - 1$), and the observations T_1 to T are available for the out of sample estimation, i.e. a total holdout sample of T - ($T_1 - 1$).

The MSE value would be compared with those of other models for the same data and forecast period and the model with the lowest value of the error measure would be argued to be most accurate (Brooks, 2002).

2) Theil's *U*-Statistic

A popular criterion used is the Theil (1966) *U*-statistic which metric is designed as follows:

$$U=\sqrt{\frac{\sum_{t=1}^{n-1}(\frac{\hat{y}_{t+1}-y_{t+1}}{y_t})^2}{\sum_{t=1}^{n-1}(\frac{y_{t+1}-y_t}{y_t})^2}} \tag{31}$$

Theil's *U* statistic is a measure of the degree to which one time series (X_i) differs from another (Y_i) (http://www.uvm.edu/giee/AV/Spatial_Modeling_Book/4/node33.html). A U-statistic of one implies that the model under consideration and the benchmark model are equally (in)accurate, while the value of less than one implies that the model is superior to the benchmark and vice versa of U > 1 (Brooks, 2002).

III. Results

1. Maximum Entropy (ME) Model

The GAMS (General Algebraic Modeling System) Program is used to solve the numerical optimization problems using non-linear programming. Table 3 below shows the parameters estimated by the model for use in the analysis.

〈Table 3〉 Estimated parameters in ME Model

A	α	β	r	K
0.165	0.636	0.39	0.422	240,500

Integrating the estimated parameters in the Table 3 above into estimated equations comes up with the following:

$$C_t = 0.165\ E_t^{0.636}\ X_t^{0.39} \tag{32}$$

$$X_{t+1} - X_t = 0.422(1 - \frac{X_t}{240{,}500}) - C_t \tag{33}$$

From the results of equation (32) the Kapenta fishery demonstrates increasing returns to effort and stock since the sum of the exponents, α and β, is 1.026. The effort elasticity of catch, α, suggests that a 10 percent increase in effort will increase the Kapenta catch by 6.4 percent. The stock elasticity, β, is estimated to be 0.39 implying that doubling the stock size would result in a 39% increase in catch (holding all others constant). The technical efficiency (defined as the improvement of fishing gear to improve fishing yields (Sun, 1999)) is low with a value of 0.165. This means less effort could be employed to realize the same level of catch using more efficient (technical) methods.

〈Table 4〉 Estimated annual stock of Kapenta in Lake Kariba by the ME model

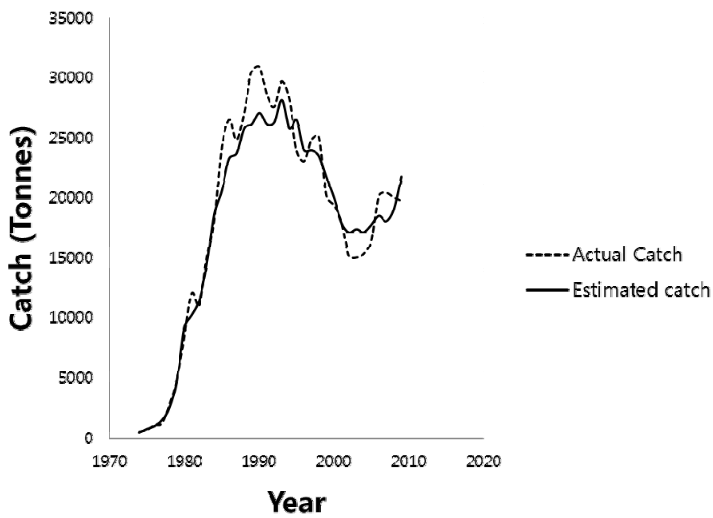
Year	Estimated Probabilities			Estimated stock (tons)	ABC (tons)
	X_t = 0	X_t = 92,500	X_t = 185,000		
1974	0.836	0.164	0	15,170	2,034
1975	0.805	0.195	0	18,037.5	2,671
1976	0.769	0.231	0	21,367.5	3,410
1977	0.73	0.27	0	24,975	4,212
1978	0.629	0.33	0.041	38,110	7,129
1979	0.534	0.33	0.136	55,685	11,033
1980	0.513	0.33	0.157	59,570	11,895
1981	0.523	0.33	0.147	57,720	11,485
1982	0.527	0.33	0.143	56,980	11,320
1983	0.447	0.33	0.223	71,780	14,607

〈Table 4〉 Estimated annual stock of Kapenta in Lake Kariba by the ME model (Continued)

Year	Estimated Probabilities			Estimated stock (tons)	ABC (tons)
	X_t = 0	X_t = 92,500	X_t = 185,000		
1984	0.311	0.33	0.359	96,940	20,195
1985	0.186	0.33	0.484	120,065	25,332
1986	0.072	0.33	0.598	141,155	25,373
1987	0.164	0.33	0.506	124,135	25,373
1988	0.000	0.313	0.687	156,047.5	25,373
1989	0.000	0.33	0.67	154,475	25,373
1990	0.000	0.33	0.67	154,475	25,373
1991	0.130	0.33	0.54	130,425	25,373
1992	0.225	0.33	0.445	112,850	23,729
1993	0.099	0.33	0.571	136,160	25,373
1994	0.208	0.33	0.462	115,995	24,428
1995	0.292	0.33	0.378	100,455	20,976
1996	0.347	0.33	0.323	90,280	18,716
1997	0.367	0.33	0.303	86,580	17,894
1998	0.424	0.33	0.246	76,035	15,552
1999	0.481	0.33	0.189	65,490	13,210
2000	0.536	0.33	0.134	55,315	10,950
2001	0.58	0.33	0.09	47,175	9,142
2002	0.622	0.33	0.048	39,405	7,417
2003	0.663	0.33	0.007	31,820	5,732
2004	0.651	0.33	0.019	34,040	6,225
2005	0.625	0.33	0.045	38,850	7,293
2006	0.597	0.33	0.073	44,030	8,444
2007	0.566	0.33	0.104	49,765	9,718
2008	0.603	0.33	0.067	42,920	8,197
2009	0.575	0.33	0.095	48,100	9,348

The logistic growth function in ME model is used to estimate MSY of 25,372 tons and the biomass at MSY (B_{MSY}) of 120,250 tons. The ME model estimates an effort at MSY to be 109,731 fishing nights suggesting that the current level of effort of 150,332 fishing nights is above that required to achieve MSY levels. Table 4 shows the estimates of annual biomass by the ME model and annual ABC by the Tier methods for ABC. It estimates that there is a steady decline in stock from 1988, when the stock was estimated to be at its maximum (156,047 tons), and that the current stock was seriously decreased until less than one third of the maximum, 48,100 tons. The estimated catch of the ME model was calculated using equation (32). Figure 2 shows that the ME model approximated the actual catch fairly well and this is supported by the value of the Theil's *U*-statistic of 0.46, which shows that the estimated catch and the actual catch are similar.

〈Figure 2〉 Estimated catch by the ME Model in relation to actual catch



2. Analytical Model

1) Mortality

The estimates of mortality were calculated using three different methods which are represented in Table 5 below. The fishing mortality was calculated to be 0.927/year for 2009 which is higher than that of the 1982 levels.

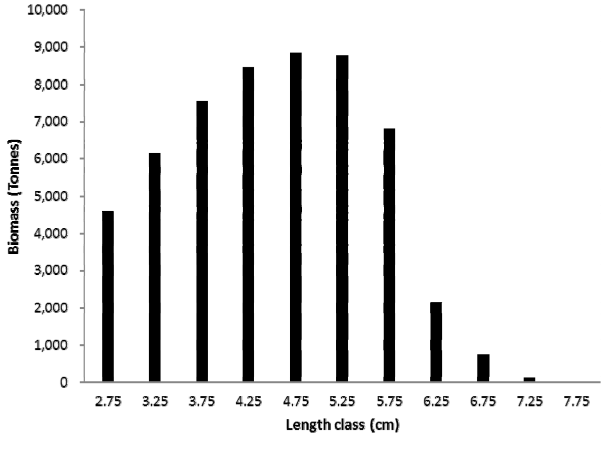
〈Table 5〉 Fishing mortality parameters for *Limnothrissa miodon* in Lake Kariba in 1982

Model/parameters	M (/year)	F (/year)	Z (/year)
Zhang & Megrey (2006)	1.924	-	-
Zhang & Megrey (2010)	-	0.320	-
Total mortality (Z = F +M)	-	-	2.244

2) Biomass

The von Bertalanffy parameters from Cochrane (1984) were used in the estimation of biomass for the 1982 stock (using parameters: K = 1.74/year. L_∞ = 8.1cm and t_0 = -0.13) using the Zhang and Megrey (2010) model. A biomass of 54,272 tons was estimated for 1982. The ME model estimated a biomass of 56,980 tons is comparable to this figure. The most abundant size class is 4.75cm estimated to be 8,852 tons. The distribution of biomass by length class is shown in the Figure 3 below.

〈Figure 3〉 Biomass by class of *Limnothrissa miodon* in Lake Kariba in 1982

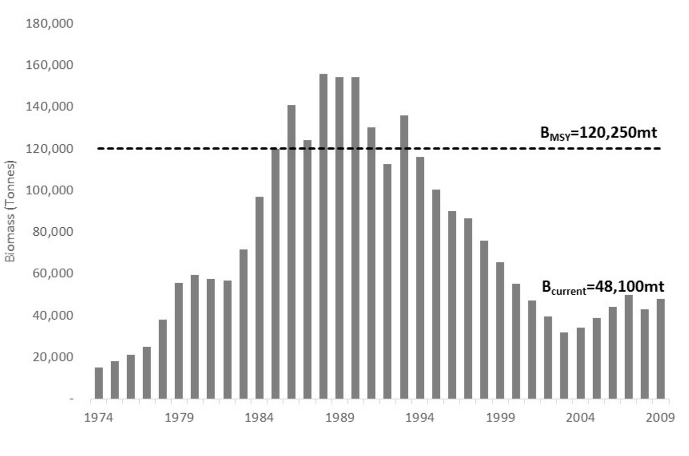


Using the F_{ABC} of 1.210/year for 1982, the spawning biomass per recruit (SPBR) was estimated to be 3.424cm and the SBPR for $F_{35\%}$ was estimated to be 3.966cm.

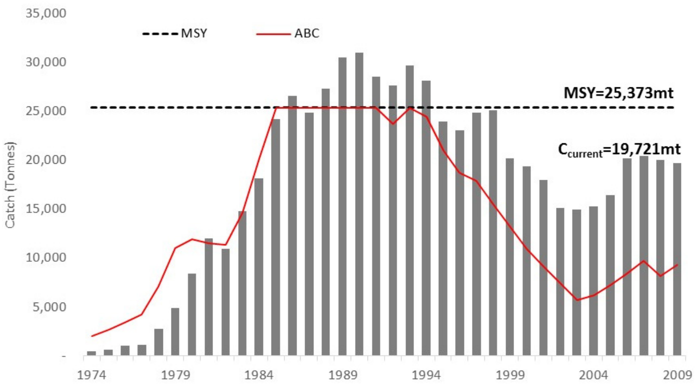
Furthermore, the estimated biomass at $F_{35\%}$ ($B_{35\%}$) was 62,509 tons which is 115% the biomass at the estimated at F_c (fishing mortality in 1982).

Figure 4 shows the estimates of annual biomass or stock and B_{MSY} (120,250 tons) by the ME model during 1974-2009. The biomass estimated by the ME model indicates the maximum value as 156,048 tons in 1988. Since then, the biomass has been continuously decreasing and it was lower than the level of B_{MSY} (120,250t) from the 2000s. Accordingly, the current biomass ($B_{current}$) was estimated to be 48,100 tons, which represents 40% of B_{MSY} as well as one third of maximum biomass (156,047 tons) in 1988. In Figure 5, an attempt was made to integrate biomass from ME model into a comparison of annual catches and ABC from Analytical model. ABC is estimated to be equal to the level of MSY (25,373 tons) when the level of biomass was higher than B_{MSY}, *vice versa* (see Table 2). As shown in Figure 5, the estimated ABCs during 1985 to 1991 are the same as MSY, but the estimated ABCs thereafter indicate quite less than MSY. Therefore, recent actual catches do not have been sustained at the level of MSY because Kapenta stock has been overexploited due to yieldings over the level of ABC from 1986. It implies that annual ABC estimated be more effective than MSY, which indicates maximum catch, as an indicator for overexploitation state.

〈Figure 4〉 Estimated biomass and B_{MSY} by the ME Model



〈Figure 5〉 Variations in annual catch and ABC



Ⅳ. Discussion and Conclusion

The Kapenta fishery was assessed using two models, one based on catch and effort and the other based on growth parameters. Due to the absence of recent data on the fishery, 1982 assessment parameters were used for the Analytical Model.

The ME model takes account of the full range of uncertainties into non-linear programming. Catch and effort are the observed variables in the model whilst the unknown parameters are probability distribution of constant, environmental carrying capacity (K), biomass and two parameters, α and β, which represent elasticity of effort and biomass respectively. The ME formulation seeks a solution that maximizes the distribution of probabilities reflecting our uncertainty about parameters subject to data consistency and normalization additivity requirements. This approach offers a method of recovering the desired parameters of stock assessment with a minimal amount of prior information when the state system is nonlinear and the state observation is noisy (Pyo, 2006). The ME model was able to estimate MSY which is higher than the prevailing catch level. Furthermore, the model estimates that there is overcapacity in the Kapenta (*Limnothrissa miodon*) Stock in Lake Kariba as evidenced by the number of fishing nights for the current fishing regime being above the estimated required fishing effort levels needed to attain MSY. The Analytical Model estimated the current fishing mortality (F) of 0.927/year, which was lower than F_{ABC} of 1.210/year and recent catch levels, which were higher than annual ABC. An attempt was made to consolidate annual biomass estimates from the ME model into the annual ABC estimates from the Analytical model. Although the level of catch is estimated to be below MSY level, it is over ABC levels from 1986. It suggests that recent actual catches do not have been sustained at the level of MSY because Kapenta stock has been overexploited due to yieldings over the level of ABC from 1986. It implies that the annual ABC estimated be more effective than MSY, which indicates maximum catch, as an indicator for overexploitation state.

In comparison, it can be seen that the Analytical Model provides a more conservative value of ABC compared to the MSY value estimated by the ME model. In order to maintain the MSY, it is necessary to establish and implement a conservative management policy such as reduction in fishing efforts as well as catches which should be below the level of ABC until the stock will be recovered sufficiently. Furthermore, the total allowable catch, control of illegal fishery, enhancement of monitoring system and on-board observer system should be taken into account to control fishing efforts and catches for Zimbabwe and Zambia simultaneously. In other way, there is a strong need for further research to be carried out on current length/weight frequencies which would provide more recent hence more accurate biological reference points for the fishery.

[References]

Beverton, R. J. H. and S. Holt, *On the Dynamics of Exploited Fish Populations*, Chapman & Hall Fish and Fisheries Series, Vancouver, Canada, 1957.

Brooks C., *Introductory Econometrics for Finance*, Cambridge university Press, 2002.

Chifamba, P. C., Daily Rings on Otholiths as a Method for Ageing the Sardine *Limnothrissa miodon* in Lake Kariba. *Trans. Zimbabwe Sci. Ass*. 66, 1992.

Cochrane, K. L., "The Influence of Food Availability, Breeding Seasons and Growth on Commercial Catches of *Limnothrissa miodon* (Boulenger) in Lake Kariba," *J. Fish Biol*. Vol. 24, 1984, pp. 623~635.

Golan, A, G. Judge, and D. Miller, *A Maximum Entropy Econometrics*, John Wiley and Sons. 1996b.

Golan, A., G. Judge, and L. Karp, "A Maximum Entropy Approach to Estimation and Inference in Dynamic Models or Counting Fish in the Sea using Maximum Entropy," *Journal of Economic Dynamics and Control*, Vol. 20, 1996a, pp. 559~582.

Hilborn, R. and C. J. Walters, "A General Model for Simulation of Stock and Fleet Dynamics in Spatially Heterogeneous Fisheries," *Canadian Journal of Fisheries and Aquatic Sciences*, Vol. 44, No. 7, 1987, pp. 1366~1369.

http://www.uvm.edu/giee/AV/Spatial_Modeling_Book/4/node33.html

Marshall B. E., "Growth and mortality of the Introduced Lake Tanganyika clupeid," *Limnothrissa miodon*, in Lake Kariba. *J. Fish Biol*. Vol. 31, 1987, pp. 603~615.

Pascoe S., A Bioeconomic Analysis of the UK Fisheries of the English Channel. PhD Thesis, University of Portsmouth, UK, 1998.

Pyo, H. D., "A Comparative Analysis of Surplus Production Models and a Maximum Entropy Model for Estimating the Anchovy Stock in Korea," *Jour. Fish. Mar. Sci. Edu*. Vol. 18, No. 1, 2006, pp. 19~30.

Quinn, T. D., and R. B. Deriso RB, *Quantitative Fish Dynamics*, Oxford University Press. New York, 1999, p. 542.

Shannon, C. E., "A Mathematical Theory of Communication," *Bell System Technical Journal*, Vol. 27, 1948, pp. 379~423.

Sun C. H., "Optimal Number of Fishing Vessels for Taiwan's Offshore Fisheries: A Comparison of Different Fleet Size Reduction Policies," *Marine Resource Economics*, Vol. 13, 1999, pp. 275~288.

Theil, H., *Applied Economic Forecasting*, North-Holland. Amsterdam, 1966.

Tresierra, A. and Z. Culquichicón, *Fisheries Biology*. Trujillo. Perú, 1993, p. 432.

Vignaux, M., G. A. Vignaux, S. Lizamore, and D. Gresham, "Fine-scale Mapping of Fish Distribution From Commercial Catch and Effort Data Using Maximum Entropy Tomography," *Canadian Journal of Fisheries and Aquatic Sciences*, Vol. 55, No. 5, 1998, pp. 1220~1227.

Xiao Y., "Subtleties In, And Practical Problems With, The Use of Production Models in Fish Stock Assessment," *Fisheries Research*, Vol. 33, 1997, pp. 17~36.

Zhang, C. I. and B. A. Megrey, "A revised Alverson and Carney Model for Estimating the Instantaneous Rate of Natural Mortality," *Transactions of the American Fisheries Society*, Vol. 135, 2006, pp. 620~633.

Zhang, C. I. and B. A. Megrey, "A Simple Biomass-Based Length-Cohort Analysis for Estimating Biomass and Fishing Mortality," *Transactions of the American Fisheries Society*, Vol. 139, 2010, pp. 911~924.

Zhang, C. I. and J. B. Lee. "Stock Assessment and Management Implications of Horse Mackerel (Trachurus japonicus) in Korean waters, based on the relationships between recruitment and the ocean environment," *Progress in Oceanography*, Vol. 49, 2001, pp. 513~537.